

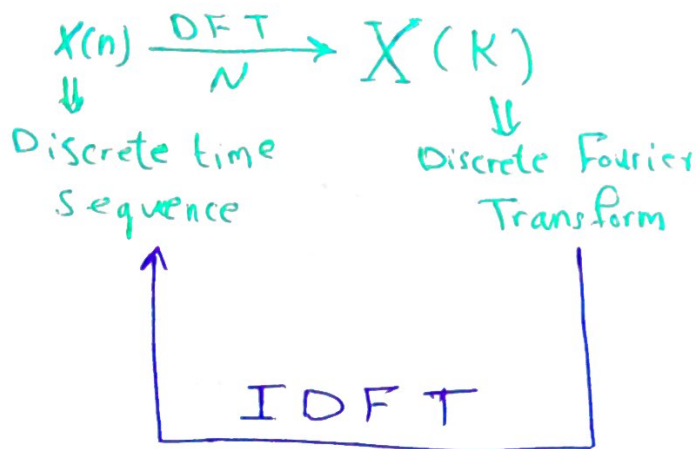
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الأربعاء

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محاضرة [6]

Inverse Discrete Fourier Transform (IDFT)



$$X(K) \xrightarrow[\frac{1}{N}]{\text{IDFT}} x(n)$$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} K n}$$

$$X(K) \xrightarrow[\frac{1}{N}]{\text{IDFT}} x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi}{N} K n}$$

notes:-

* If the discrete time sequence $x(n)$ is periodic every N sample, then the discrete Fourier transform is also periodic

* If $X(K)$ is periodic, then the discrete time sequence $x(n)$ is also periodic.

$$X(K) \xrightarrow[\frac{1}{N}]{\text{IDFT}} x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi}{N} K n}$$

\downarrow required the discrete time seq. $x(n)$
 \uparrow given DFT $X(K)$

$$W_N = e^{-j \frac{2\pi}{N}} \Rightarrow \text{twiddle Factor}$$

$$X(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn}$$

$$\Rightarrow X_N = \frac{1}{N} [W_N^*] X_N$$

$$X_N = \begin{bmatrix} x(n=0) \\ x(n=1) \\ x(n=2) \\ \vdots \\ x(N-1) \end{bmatrix} \rightarrow \text{discrete time sequence in vector form}$$

$$X_N = \begin{bmatrix} X(k=0) \\ X(k=1) \\ \vdots \\ X(k=N-1) \end{bmatrix} \rightarrow \text{DFT in vector form}$$

$$[W_N^*] = \begin{pmatrix} W_N^{-kn} \end{pmatrix}_{N \times N}$$

for $N=3$

$$[W_3^*] = \begin{matrix} & \begin{matrix} n=0 & n=1 & n=2 \end{matrix} \\ \begin{matrix} k=0 \\ k=1 \\ k=2 \end{matrix} & \begin{pmatrix} W_3^0 & W_3^0 & W_3^0 \\ W_3^0 & W_3^{-1} & W_3^{-2} \\ W_3^0 & W_3^{-2} & W_3^{-4} \end{pmatrix} \end{matrix}$$

هناك نفس W_3 مع تبديل اتجاهات الأجزاء التخيلية

$$[W_3^*] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 + j\sqrt{3}/2 & -0.5 - j\sqrt{3}/2 \\ 1 & -0.5 - j\sqrt{3}/2 & -0.5 + j\sqrt{3}/2 \end{bmatrix}$$

$$\text{For } [W_4^*] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

Example:- Find IDFT of $X(K) = \{6, -2+j2, -2, -2-j2\}$

$$X_4 = \frac{1}{4} [W_4^*] X_4 \quad \swarrow \text{given}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 6 \\ -2+j2 \\ -2 \\ -2-j2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 6-2-2-j2-j2 \\ 6-2j-2+2+j2-2 \\ 6+2-j2-2+2+j2 \\ 6+2j+2+2-2j+2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 0 \\ 4 \\ 8 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

Properties of DFT

I] Periodic Property

$X(n)$ is periodic then $X(K)$ is also periodic

$$X(K) = X(K+N)$$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi K n}{N}}$$

$$\text{Put } K \rightarrow K+N$$

\Rightarrow Turn over

$$\begin{aligned}
 X(K+N) &= \sum_{n=0}^{N-1} x(n) e^{-j 2\pi (K+N)n / N} \\
 &= \sum_{n=0}^{N-1} x(n) e^{-j 2\pi K n / N} e^{-j 2\pi N n / N} \\
 &\left(\begin{aligned} e^{-j 2\pi n} &= \cos(2\pi n) - j \sin(2\pi n) \\ &= 1 \end{aligned} \right. \quad n=0, 1, 2, \dots, N-1 \\
 &= \sum_{n=0}^{N-1} x(n) e^{-j 2\pi K n / N} = X(K)
 \end{aligned}$$

2] Linearity Property

$$X_1(n) \xrightarrow[N]{\text{DFT}} X_1(K)$$

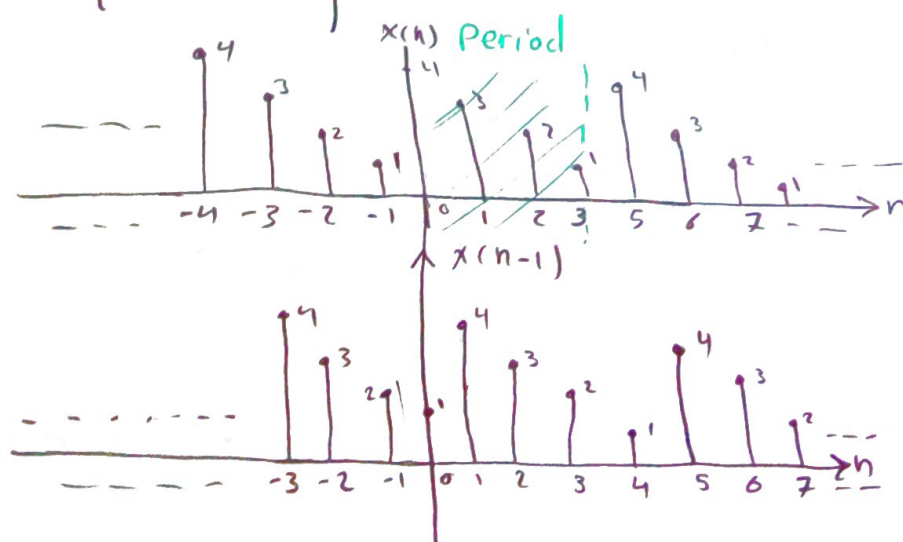
$$X_2(n) \xrightarrow[N]{\text{DFT}} X_2(K)$$

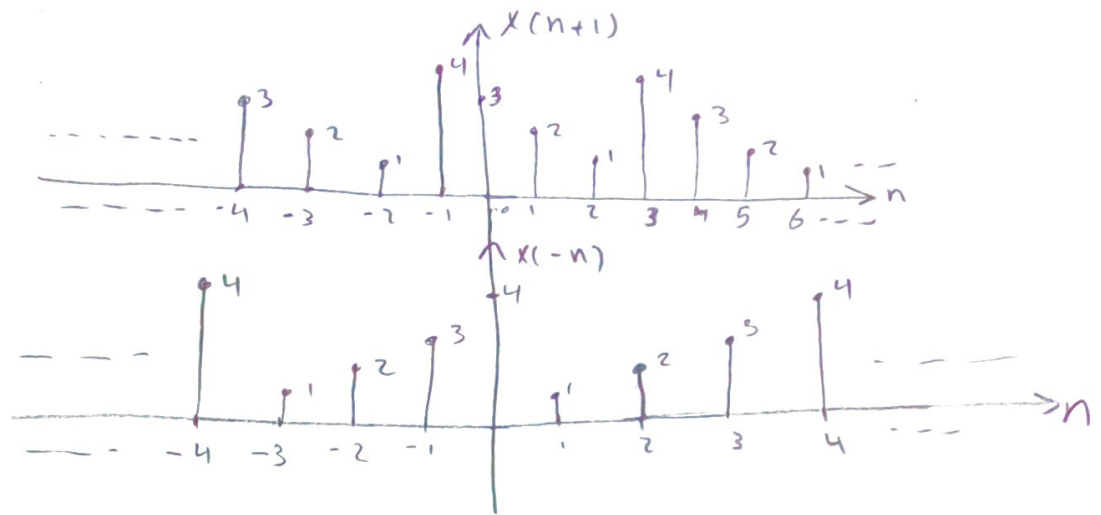
$$a_1 X_1(n) + a_2 X_2(n) \xrightarrow[N]{\text{DFT}} a_1 X_1(K) + a_2 X_2(K)$$

3] Circular Shift property

If the sequence is periodic, the shift is called circular shift.

Ex: $X(n) = \{4, 3, 2, 1\}$ is periodic sequence every 4 samples





For $0 \leq n \leq N-1$ one period

$$x(n) \{4, 3, 2, 1\} = \{x(0), x(1), x(2), x(N-1=3)\}$$

$$x((n-1))_4 \underset{\text{circular shift}}{=} \{1, 4, 3, 2\} = \{x(3), x(2), x(1), x(0)\}$$

Shift rotate to right

$$x((n+1))_4 = \{3, 2, 1, 4\}$$

Shift rotate to left

$$x((-n)) = \{4, 1, 2, 3\} \text{ Folding}$$

* انكس ما بعد أول عنصر (لبيت الأول وانكس ما يليه)

Ex: $x(n) = \{1, 0.5, 0, 1, 2\}$

- Find
- ① $x((-n))_5 \Rightarrow \{1, 2, 1, 0, 0.5\}$
 - ② $x((n-2))_5 \Rightarrow \{1, 2, 1, 0.5, 0\}$
 - ③ $x((n+2))_5 \Rightarrow \{0, 1, 2, 1, 0.5\}$
 - ④ $x((1-n))_5 \Rightarrow x((-n-1))_5 \Rightarrow \{0.5, 1, 2, 1, 0\}$
 - ⑤ $x((2-n))_5 \Rightarrow \{0, 0.5, 1, 2, 1\}$
- delay by 2